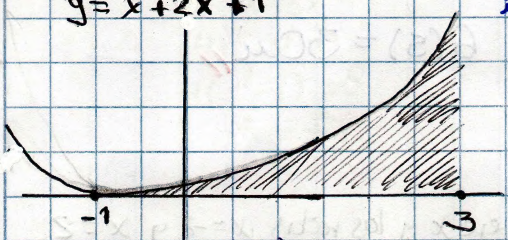
I) DETERMINAR EL ÁREA DE LA REGIÓN PARA CADA CASO, EMPLEANDO DEFINICIÓN DE INTEGRAL DEFINIDA:

1) La región plana acotada por $y = x^2+2x+1$, el eje x y las rectas $x=-1$ y $x=3$

$y = x^2+2x+1$



i) $\lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x = \int_{-1}^{3} x^2+2x+1\,dx$

ii) $\Delta x = \frac{b-a}{n} = \frac{4}{n}$

iii) $x_i = a + i\Delta x = i\frac{4}{n} - 1$

iv) $\lim_{n\to\infty}\sum_{i=1}^{n} f\left(\frac{4i}{n}-1\right)\frac{4}{n} = \lim_{n\to\infty}\sum_{i=1}^{n}\left[\left(\frac{4i}{n}-1\right)^2 + 2\left(\frac{4i}{n}-1\right)+1\right]\frac{4}{n}$

• $\left(\frac{4i}{n}-1\right)^2 = \frac{16i^2}{n^2} - \frac{8i}{n} + 1$

• $2\left(\frac{4i}{n}-1\right) = \frac{8i}{n} - 2$

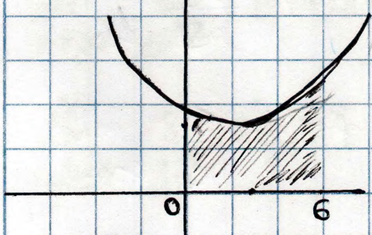
$\lim_{n\to\infty}\frac{4}{n}\sum_{i=1}^{n}\left(\frac{16i^2}{n^2} - \cancel{\frac{8i}{n}} \cancel{+1} \cancel{-\frac{8i}{n}} \cancel{-2} \cancel{+1}\right) = \lim_{n\to\infty}\frac{4}{n}\sum_{i=1}^{n}\frac{16i^2}{n^2}$

$\lim_{n\to\infty}\frac{64}{n^3}\sum_{i=1}^{n} i^2 = \lim_{n\to\infty}\frac{64}{n^3}\frac{n(n+1)(2n+1)}{6} = \frac{64}{6}\lim_{n\to\infty}\frac{2n^3+3n^2+n}{n^3}$

$\frac{64}{6}\lim_{n\to\infty}\frac{\cancel{n^3}\left(2+\frac{3}{n}+\frac{1}{n}\right)}{\cancel{n^3}} = \frac{64}{6}(2) = \frac{64}{3}u$ //

2) La región plana acotada acotada por $y=(x-3)^2+2$, el eje x y las rectas $x=0$ y $x=6$

$y=(x-3)^2+2$



i) $\lim_{x\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x = \int_{0}^{6}(x-3)^2+2\,dx$

ii) $\Delta x = \frac{b-a}{n} = \frac{6}{n}$ $\wedge$ $x_i = a + i\Delta x = \frac{6i}{n}$

iii) $\lim_{x\to\infty}\sum_{i=1}^{n} f\left(\frac{6i}{n}\right)\frac{6}{n} = \lim_{x\to\infty}\frac{6}{n}\sum_{i=1}^{n}\left(\frac{6i}{n}-3\right)^2+2$

• $\sum_{i=1}^{n}\frac{1}{n^2}(6i-3n)^2+2 = \sum_{i=1}^{n}\frac{36i^2}{n^2} - \frac{36in}{n^2} + \frac{9\cancel{n^2}}{\cancel{n^2}} + 2 = \sum_{i=1}^{n}\frac{36i^2}{n^2} - \frac{36i}{n} + 11$

• $\frac{36}{n^2}\sum_{i=1}^{n} i^2 - \frac{36}{n}\sum_{i=1}^{n} i + 11\sum_{i=1}^{n} = \frac{\cancelto{6}{36}}{\cancel{n^2}}\left(\frac{\cancel{n}(n+1)(2n+1)}{\cancel{6}}\right) - \frac{\cancelto{18}{36}}{\cancel{n}}\frac{\cancel{n}(n+1)}{\cancel{2}_1} + 11n$

• $\frac{6(n+1)(2n+1) - 18n(n+1) + 11n^2}{n}$